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LOCALIZED OPTICAL MODES IN OPTICS OF PHOTONIC LIQUID CRYSTALS

ЛОКАЛИЗОВАННЫЕ ОПТИЧЕСКИЕ МОДЫ В ОПТИКЕ ФОТОННЫХ ЖИДКИХ КРИСТАЛЛОВ

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The localized optical modes in chiral liquid crystals are theoretically investigated. Since the cholesteric liquid crystals (CLC) demonstrate common for all chiral liquid crystals optical properties the studying of the problem is performed for the certainty on the CLCs example. A brief survey of the recent experimental and theoretical results on the low threshold distributed feedback (DFB) lasing in chiral liquid crystals as well as new original theoretical data on the localized optical modes in CLC (edge (EM) and defect (DM)) related to the anomalously strong absorption (amplification) of light at the frequencies of EM and DM are presented. An analytic approach to the theory of the EM and DM optics in CLC is developed. The dispersion equations determining connection of the EM and DM frequencies with the CLC layer parameters and other parameters of the defect structure (DMS) are obtained. Analytic expressions for the transmission and reflection coefficients of the DMS are presented and analyzed. As specific cases DMS with an active defect layer are considered, i.e. the DMS with birefringent, absorbing and amplifying defect layers in a perfect CLC structure.

Key words: *chiral liquid crystals, edge modes, defect modes, low threshold DFB lasing.*

Теоретически исследованы локализованные оптические моды в оптике фотонных жидких кристаллов на примере холестерических жидких кристаллах (ХЖК), поскольку они проявляют общие для всех хиральных жидких кристаллов оптические свойства. Приведен краткий обзор последних экспериментальных и теоретических результатов по низкопороговой лазерной РОС (с распределенной обратной связью) генерации в хиральных жидких кристаллах, а также новые оригинальные теоретические результаты по локализованным оптическим модам в ХЖК (краевым (EM) и дефектным (DM) модам), относящиеся к эффектам аномально сильного поглощения (усиления) света на частотах EM и DM. Развит аналитический подход к оптике ХЖК в области частот EM и DM мод. Получены дисперсионные уравнения для EM и DM, устанавливающие связи частот EM и DM с параметрами холестерических слоев и параметрами дефектных структур (DMS). Представлены и проанализированы аналитические выражения для коэффициентов пропускания и отражения DMS. В качестве конкретных примеров рассмотрены DMS с активными дефектными слоями, т. е. DMS с двулучепреломляющими, поглощающими и усиливающими слоями в совершенной ХЖК структуре.

Ключевые слова: *хиральные жидкие кристаллы, краевые моды, дефектные моды, низкопороговая лазерная РОС генерация.*

1. Introduction

Recently there was a very intense activity in the field of localized optical modes, in particular, edge (EM) and defect (DM) modes in chiral liquid crystals (CLC) mainly due to the possibilities to reach a low lasing threshold for the mirrorless distributed feedback (DFB) lasing [1–4] in chiral liquid crystals. The EM and DM existing as a localized electromagnetic eigen state with its frequency close to the forbidden band gap or in the forbidden band gap, respectively, were investigated initially in the periodic dielectric structures [5]. The corresponding EM and DM in chiral liquid crystals, and more general in spiral media, are very similar to the EM and DM in one-dimensional scalar periodic structures. They reveal abnormal reflection and transmission [1, 2] and allow DFB lasing at a low lasing threshold [3]. Almost all studies of the EM and DM in chiral and scalar periodic media were performed by means of a numerical analysis with the exceptions [6, 7], where the known exact analytical expression for the eigen waves propagating along the helix axis [8, 9] were used for a general studying of the DM. The used in [6, 7] approach looks as a very fruitful one because it allows to reach easy understanding of the DM and EM physics and it is why it deserves further implementation in the studying of the EM and DM at the base of well developed theory of the CLC optics [10–13]. In the present paper analytical solutions of the EM and DM mode (associated with an insertion of a layer in the perfect cholesteric structure) are presented and some limiting cases simplifying the problem are considered. Because the cases of EM and DM at an isotropic defect structure (DMS) were already studied (see, [14–16]) we shall present here the results related to a DMS with an active defect layer (birefringent, absorbing or amplifying).

Recently a lot of new types of defect layer were studied [17–23]. The consideration below will be limited by a birefringent or absorbing (amplifying) layer inserted in a chiral liquid crystal. The reason for that is connected as with the experimental [22, 24] and theoretical [23, 25, 26] researches of the DFB lasing in CLC where a defect layer is birefringent or absorbing (amplifying) so with a general idea that the unusual properties of DM manifest themselves most clearly just at the middle of DMS, i.e. at defect layer where intensity of the DM field reaches its maximum. The analytic approach in studying of a DMS with a birefringent or absorbing (amplifying) defect layer is very similar to the previously performed DM studies for isotropic defect layer [15, 27], so we shell present below the final results of the present investigation sending the readers for the investigation details to references [15, 27].

In the following sections the dispersion equation and analytical expressions for transmission and reflection coefficients for the defect mode associated with an insertion of a birefringent or absorbing (amplifying) defect layer in the perfect cholesteric structure are presented for light propagating along the helical axes.

2. Defect mode at birefringent defect layer

Nonabsorbing CLC layers

To consider the DM associated with an insertion of a birefringent layer in the perfect cholesteric structure we have *to* solve Maxwell equations and a boundary problem for electromagnetic wave propagating along the cholesteric helix for the layered structure depicted at fig.1. Exploiting results obtained in [28] (and using the same simplifications) one easily gets the results related a birefringent layer. For example, if one neglects the multiple scattering of light of nondifracting in CLC polarization the transmission $|\mathrm{T(d,L)}|^2$ and reflection $|\mathrm{R(d,L)}|^2$ intensity coefficients (of light of diffracting circular polarization) for the whole structure may be presented in the following form:

$$|T(d,L)|^2 = |[T_e T_d M(k,d,\Delta n)(\sigma_e \sigma_{ed}^*)]/[1 - M^2(k,d,\Delta \mathrm{n})(\sigma_r \sigma_{ed}^*)^2 R_d R_u]|^2, \quad (1)$$

$$|R(d,L)|^2 = |\{R_e + R_d T_e T_u M^2(k,d,\Delta n)|(\sigma_e \sigma_{ed}^*)|^2 / [1 - M^2(k,d,\Delta n)(\sigma_r \sigma_{ed}^*)^2 R_d R_u]\}|^2, \quad (2)$$

where L is the CLC layer thickness, $R_e(T_e)$, $R_u(T_u)$ and $R_d(T_d)$ [16, 30] are the amplitude reflection (transmission) coefficients of the individual CLC layer (see fig. 1) for the light of diffracting polarization incident at the outer (top) layer surface, for the light incidence at the inner top CLC layer surface from the inserted defect layer and for the light incidence at the inner bottom CLC layer surface from the inserted defect layer, respectively, σ_e, σ_r and σ_{ed} are the polarization vectors of light exiting the CLC layer inner surface, reflected at the inner bottom CLC layer surface at the incidence from the inserted defect layer and of light whose some polarization vector σ_{ed} transforms to the polarization vector σ_e at crossing the birefringent defect layer of thickness d, respectively, Δn is the difference of two refractive indexes in the birefringent defect layer and $M(k,d,\Delta n)$ is the phase factor related to light single propagation in a birefringent defect layer.

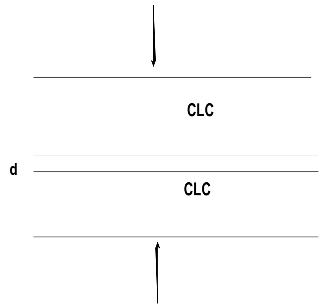


Fig. 1. Schematic of the DMS with a birefringent defect layer

The expressions for the amplitude transmission T and reflection R coefficients for circularly polarized light (of diffracting circularl polarization) incident at a CLC layer of thickness L are given by the following formula [14—16]:

$$R = i\delta \sin qL / \{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\},$$
$$T = \exp[i\kappa L](q\tau/\kappa^2)/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\}, \quad (3)$$

where $q = \kappa\{1 + (\tau/2\kappa)^2 - [(\tau/\kappa)^2 + \delta^2]^{1/2}\}$, $\varepsilon_0 = (\varepsilon_{||} + \varepsilon_{\perp})/2$, $\delta = (\varepsilon_{||} - \varepsilon_{\perp})/(\varepsilon_{||} + \varepsilon_{\perp})$ is the dielectric anisotropy, and $\varepsilon_{||}$, and $\varepsilon_{\perp}$ are the local principal values of the CLC dielectric tensor [10–12], $\kappa = \omega\varepsilon_0/c$, c is the speed of light, $\tau = 4\pi/p$ and p is the cholesteric pitch.

The corresponding polarization vectors in (1, 2) may be found (see [11, 12]) as well as the polarization vector σ_{ed} may be easily calculated if the d, and Δn are known. The calculation of the reflection and transmission coefficients according (1, 2) is performable analytically in the general case, however, it is rather cumbersome. It is why below will be studied in details the case of a low birefringence.

Under the mentioned above simplification and the assumption that the refractive indexes of the DMS external media coincides with the average CLC refractive index and the average refractive indexes of defect layer the refractive indexes of defect layer may be given by the formulas

$$n_{max} = n_0 + \Delta n/2, \quad n_{min} = n_0 - \Delta n/2, \quad (4)$$

where n_0 coincides with the average CLC refractive index and Δn is small i. e. $\Delta n/n_0 < \delta$. The phase difference of two beam component with different eigen polarization at the defect layer thickness is $\Delta\varphi = \Delta n \mathrm{k} d/n_0$.

Finally, one gets the following expressions for reflection and transmission coefficients of light with a diffracting polarization for the incident beam with diffracting polarization in the case of low birefringence:

$$|T(d,L)|^2 = |[T_e T_d \exp[ikd]\cos(\Delta\varphi/2)]/[1-\exp[i2kd]\cos^2(\Delta\varphi/2)R_d R_u]|^2, \tag{5}$$

$$|R(d,L)|^2 = |\{R_e + R_d\, T_e T_u \exp[i2kd]\cos^2(\Delta\varphi/2)/[1-\exp[i2kd]\cos^2(\Delta\varphi/2)R_d R_u]\}|^2 \tag{6}$$

The calculations results for transmission $|T(d,L)|^2$ coefficients of light of diffracting polarization for the case of low birefringence are presented at figs. 2 for various values of the birefringent phase factor Δφ. Figs. 2 show that at low values of phase shift between eigen waves at their crossing the defect layer (Δφ < π/2) the shape of transmission curve is very similar to those for DMS with an isotropic defect layer.

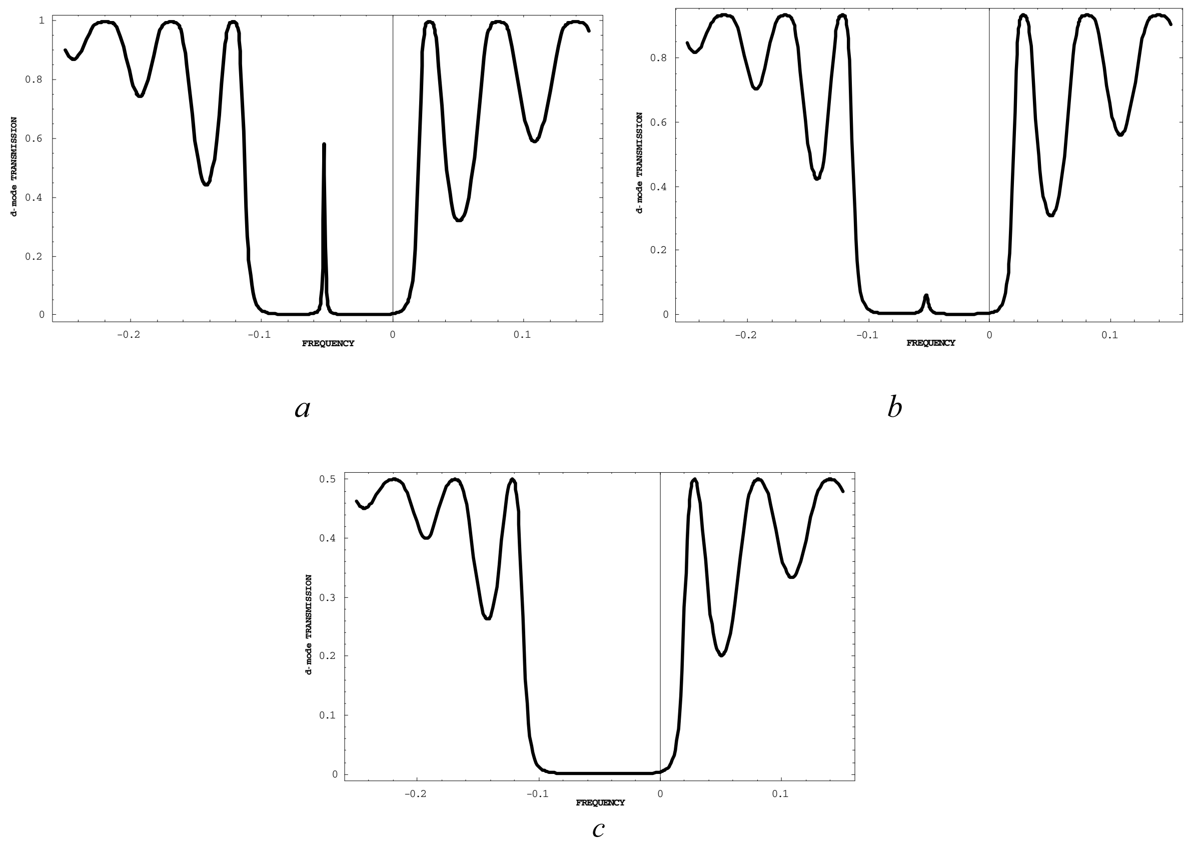


Fig. 2. Calculated diffracting polarization intensity transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequencyν (Here and at all other figures $\nu = \delta[2(\omega-\omega_B)/(\delta\omega_B) - 1)]$, δ = 0,05 and N = 33 is the director half-turn number at the CLC layer thickness L) for a diffracting incident polarization at the birefringent phase shift at the defect layer thickness Δφ = π/20 (*a*), Δφ = π/6 (*b*), Δφ = π/2 (*c*), at d/p = 0,25

However, at approaching $\Delta\varphi$ to $\pi/2$ (see figs. 2) typical for an isotropic defect layer increase of transmission at the defect mode frequency gradually disappears and at $\Delta\varphi = \pi/2$ (fig. 2, *c*) does not appear at all. It is well known [13] that the position of the DM frequency in the stop band is determined by the frequency of the transmission (reflection) coefficient maximum (minimum) so the performed calculation of the transmission spectra (figs. 2) determine a real component of the DM frequency. However because DM is a quasistationary mode an imaginary component of the DM frequency is not zero [15, 27]. A direct way to find the imaginary component of the DM frequency is a solving of the dispersion equation. The dispersion equation if the multiple scattering of nondifracting polarization light being neglected is presented by the following relationship:

$$\{M^2(k,d,\Delta n)\sin^2 qL - \exp(-i\tau L)[(\tau q/\kappa^2)\cos qL + i((\tau/2\kappa)^2+(q/\kappa)^2-1)\sin qL]^2/\delta^2]\}=0. \quad (7)$$

Amplifying and absorbing CLC layers

As the experiment [3] and the theory [15, 27] show unusual optical properties of DMS at the defect mode frequency ω_D may be effectively used for enhancement of the DFB lasing. For studying how the birefringent defect layer does influence anomalously strong amplification and absorption effects we assume, as was done in [15, 27], that the average dielectric constant of CLC has an imaginary addition, i. e. $\varepsilon = \varepsilon_0(1 + i\gamma)$ (note, that at real situations $|\gamma| \ll 1$). The value of γ ensuring the onset of the lasing threshold may be found from solution of the dispersion equation (7). Another option (see [15, 27]) is studying of reflection and transmission coefficients (5, 6) as a function of γ.

For amplifying CLC the value of γ corresponding to the divergence of DMS reflection and transmission coefficients just determines the solution of the dispersion equation (7) and also determines the threshold DFB lasing gain in the DMS (see [15, 27]). So, there is an option to finding the threshold value of γ by calculating the DMS reflection and transmission coefficients at varying of γ and finding its value at the points of DMS reflection and transmission coefficients divergence.

According to the formulated approach figs. 3 presenting values of DMS transmission coefficient close to their divergence points demonstrate growth of the threshold DFB lasing gain ($|\gamma|$) with increase of the birefringent phase factor $\Delta\varphi$ and even disappearance of the divergence at defect mode frequency at $\Delta\varphi = \pi/2$.

For absorbing CLC layers in DMS the anomalously strong absorption effect reveals itself at the value of γ ensuring a maximum of the total absorption in the DMS (see [15, 27]). For finite thicknesses of CLC layers L the DM frequency ω_D occurs to be a complex quantity which may be found by a numerical solution of eq. (7). For a very small values of the parameter γ the reflection and transmission spectra of MDS with absorbing (amplifying) CLC layers are similar to the studied in [15, 27] spectra (see figs. 2). In particular, positions of dips in reflection and spikes in transmission inside the stop-band just correspond to $Re[\omega_D]$ and this observation is very useful for numerical solution of the dispersion equation. What is concerned of the DM life-time it reduces for absorbing CLC layers compared to the case of nonabsorbing CLC layers [15, 27]. However, as the above formulas and figures show even for nonabsorbing CLC layers a reducing of the DM life-time occurs due to a birefringent defect layer [28].

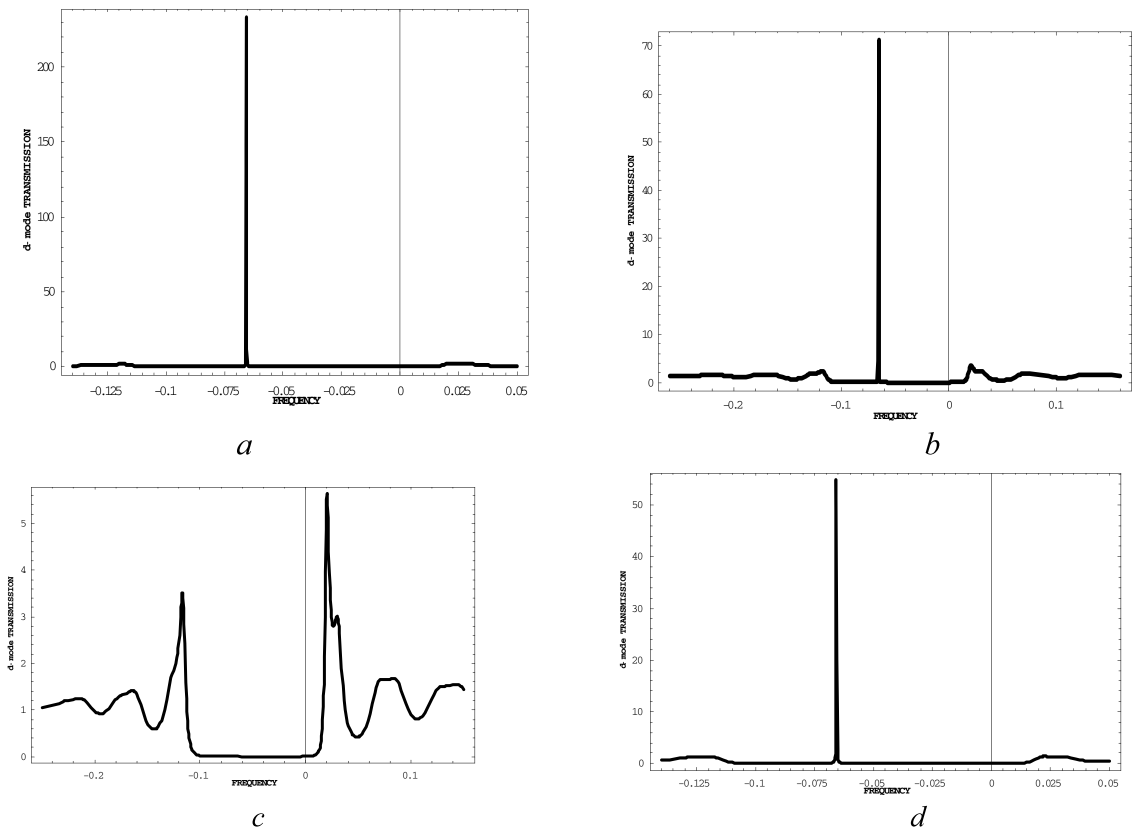


Fig. 3. Calculated intensity transmission coefficients at a low birefringent defect layer for an amplifying CLC layer versus the frequency close to their divergence points for diffracting incident polarization at
$\Delta\varphi = \pi/8$, $\gamma = -0{,}00150$ (*a*), $\Delta\varphi = \pi/6$, $\gamma = -0{,}002355$(*b*),
$\Delta\varphi = \pi/2$, $\gamma = -0{,}004500$ (*c*), $\Delta\varphi = 0$, $\gamma = -0{,}000675$ (*d*); d/p = 2,25

3. Absorbing (amplifying) isotropic defect layer

The studying of the defect mode associated with an insertion of an absorbing (amplifying) isotropic layer (fig. 1) is performed in the same manner as above (see also [15, 16, 27]). The transmission $|T(d,L)|^2$ and reflection $|R(d,L)|^2$ intensity coefficients (of light of diffracting circular polarization) for the whole structure may be presented in the following form:

$$|T(d,L)|^2 = |[T_e T_d \exp(ikd(1+ig))]/[1-\exp(2ikd(1+ig))\,R_d R_u]|^2, \quad (8)$$

$$|R(d,L)|^2 = |\{R_e + R_u\, T_e T_u \exp(2ikd(1+ig))/[1-\exp(2ikd(1+ig))R_d R_u]\}|^2, \quad (9)$$

where $R_e(T_e)$, $R_u(T_u)$ and $R_d(T_d)$ are determined above. The factor $(1 + ig)$ is related to the defect layer only and corresponds to the dielectric constant of the defect layer having the form $\varepsilon_0(1 + 2ig)$ with a small g being positive for an absorbing defect layer and negative for an amplifying one.

The defect mode frequency ω_D is determined by the following dispersion equation:

$$\{\exp(2ikd(1 + ig))\sin^2 qL - \exp(-i\tau L)[(\tau q/\kappa^2)\cos qL + i((\tau/2\kappa)^2 + (q/\kappa)^2 - 1)\sin qL]^2/\delta^2]\} = 0. \quad (10)$$

For finite thicknesses of CLC layers L ω_D occurs to be a complex quantity which may be found by a numerical solution of (eq. 10). For very small values of the parameter g the reflection and transmission spectra of MDS with an active defect layer are similar to the studied in [15, 16, 27] spectra. In particular, positions of dips in reflection and spikes in transmission inside the stop-band just correspond to $Re[\omega_D]$. What is concerned of the DM life-time it reduces for absorbing defect layers compared to the case of nonabsorbing defect layer [15, 16, 27].

As in the case of investigated DMS with absorbing CLC layers [15, 27] in DMS with an absorbing defect layer the effect of anomalously strong absorption takes place [29]. The effect reveals itself at the DM frequency and reaches its maximum (maximum of $1 - |T(d,L)|^2 - |R(d,L)|^2$) for definite value of g which may be found using the expressions (8, 9). Figs. 4 demonstrate existence of the anomalously strong absorption effect. As follows from figs. 4 the maximum values of the anomalous absorption [11, 29] are reached for g = 0,04978 and g = 0,0008891 dependent on the correspondent defect layer thickness d.

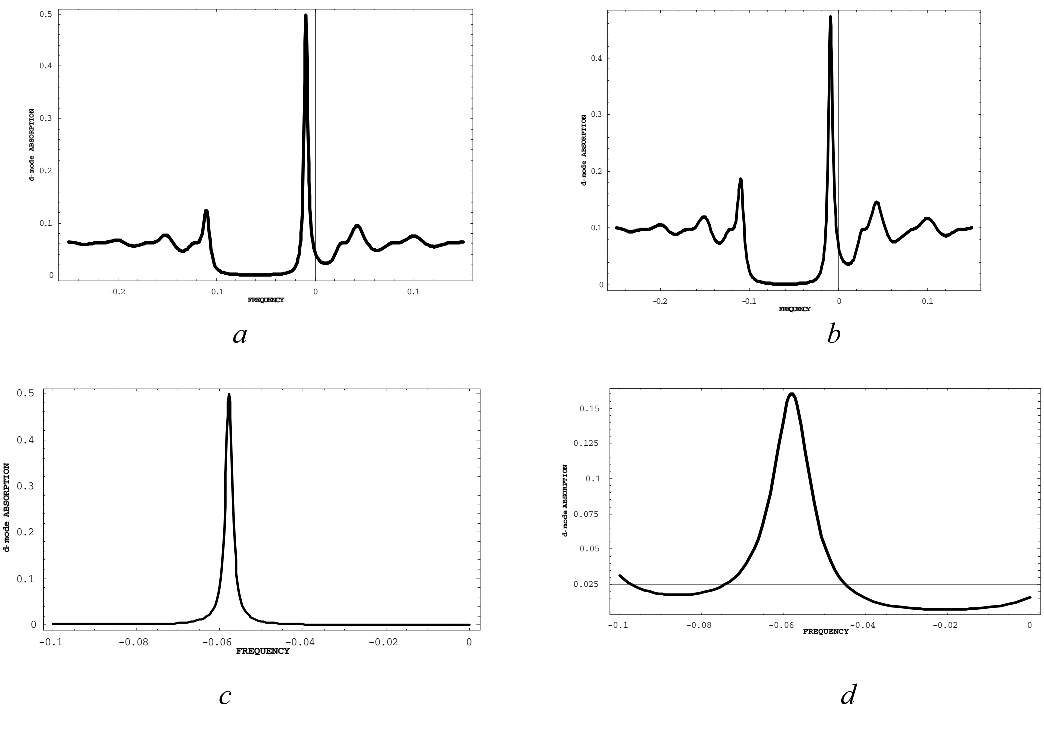


Fig. 4. Total absorption versus the frequency for an absorbing defect layer and nonabsorbing CLC layers at g = 0,04978 for d/p = 0,1 (*a*), g = 0,08 (*b*), g = 0,00008891 for d/p = 22,25(*c*), g = 0,0008891 for d/p = 22,25 (*d*)

In the case of thick CLC layers (|q|L>>1) in the DMS the g value ensuring absorption maximum may be found analytically. For the defect mode frequency ω_D in the middle of stop-band the maximal absorption corresponds to

$$g_t = (2/3\pi)(p/d)\exp[-2\pi\delta(L/p)]. \quad (11)$$

As the calculations and the formulas (11) show the gain (g) corresponding to the maximal absorption is approximately inversely proportional to the defect layer thickness *d*.
In the case of DMS with amplifying defect layer (g<0) at some value of $|g|$ divergences of reflection and transmission coefficients occur. The corresponding values of g are the gain lasing thresholds. Their values may be found numerically using the expressions (8, 9) for $|T(d,L)|^2$ and $|R(d,L)|^2$ or found approximately by plotting $|T(d,L)|^2$ and $|R(d,L)|^2$ at varying g. The second options is illustrated by figs. 5, 6, 7 where «almost divergent» values of $|T(d,L)|^2$, $|R(d,L)|^2$ or absorption $(1-|T(d,L)|^2-|R(d,L)|^2)$ are shown. The used values of *g* at figs. 5, 6, 7 are close to the threshold ones ensuring divergence of $|T(d,L)|^2$ and $|R(d,L)|^2$. The calculation results show that the minimal threshold $|g|$ corresponds to location of ω_D just in the middle of the stop-band and $|g|$ is almost inversely proportional to the defect layer thickness. Really, the figs. 5, 6 correspond to location of the defect mode frequency ω_D close to the middle point of the stop band and demonstrate decrease of the lasing threshold gain with increase of the defect layer thickness.

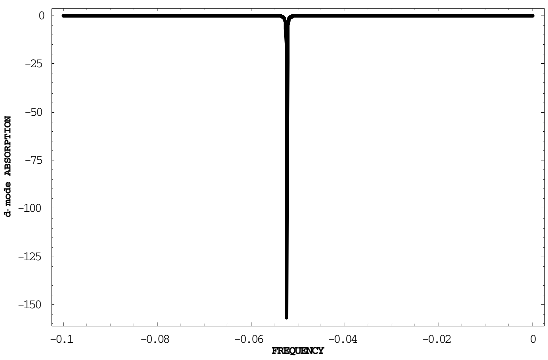
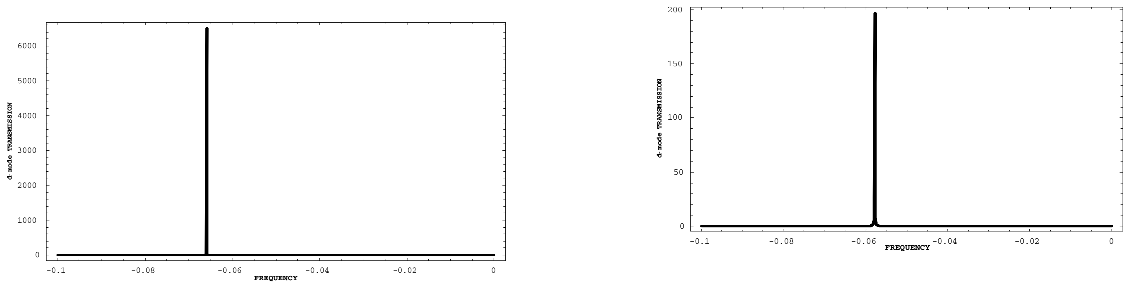


Fig. 5. Total absorption versus the frequency for amplifying defect layer and nonabsorbing CLC layers at g = – 0,0065957 for d/p = 0,25



a *b*

Fig. 6. T(d) versus the frequency for amplifying defect layer and nonabsorbing CLC layers at g = – 0,001000 for d/p = 2,25 (*a*), g = – 0,00008891 for d/p = 22,25 (*b*)

Figs. 7 correspond to location of the defect mode frequency ω_D close to the stop band edge and demonstrates increase of the lasing threshold gain with approaching the defect mode frequency ω_D to the stop band edge. The analytic approach for thick CLC layers ($|q|L>>1$) results in the similar predictions, namely, for ω_D in the middle of the stop-band the threshold value of gain is given by (11) with a negative sign of the right hand side of the expression. So, as the formula (11) shows the thinner defect layer is the higher is threshold gain g. The same result, as was mentioned above, relates also to the absorption enhancement (formula (11)). The thinner defect layer is the higher is *g* value ensuring maximal absorption.

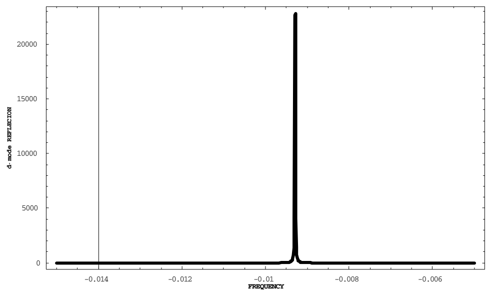


Fig. 7. R(d) versus the frequency for amplifying defect layer and nonabsorbing CLC layers at g = – 0,04978 for d/p = 0,1

4. Conclusion

The performed analytical description of the EM and DM neglecting the polarization mixing at the boundaries allows one to reveal clear physical pictures of these modes which is applicable to the DM in general. For example, more low lasing threshold and more strong absorption at the DM frequency compared to the EM frequencies are the features of any periodic media. Note, that the experimental studies of the lasing threshold [3] agree with the corresponding theoretical result obtained above. Moreover, the experiment [3] confirms also the existence of some interconnection between the gain and other CLC parameters at the threshold pumping energy for lasing at the DM and EM frequencies. For a special choice of the parameters in the experiment the obtained formulas may be directly applied to the experiment. However, in the general case one has take into account a mutual transformation at the boundaries of the two circular polarizations of opposite sense. In the general case the EM and DM field leakage from the structure is determined as well by the finite CLC layer thickness so by the leakage due to the polarization conversion. Only for sufficiently thin CLC layers or in the case of the DM frequency being very close to the stop band frequency edges the main contribution to the frequency width of the EM and DM is due to the thickness effect and the developed above model may be directly applied for the describing of the experimental data. Note also that DMS with jumps of dielectric constants at interfaces (even for an isotropic defect layer) effectively may be regarded as a DMS with an active defect layer [30]. An important result relating to the DFB lasing at DMS with active defect layer may be formulated as the following. The lasing threshold gain in defect layer decreases with the layer thickness decrease being almost inversely proportional to its thickness. The similar result relates to the effect of anomalously strong absorption phenomenon where the value of gain in the defect layer ensuring a maximal absorption is almost inversely proportional to the defect layer thickness. Note that the obtained above results are qualitatively applicable to the corresponding localized electromagnetic modes in any periodic media and may be regarded as a useful guide in the studies of the localized modes with an active defect layer in general.

It should be mentioned also that the localized DM and EM reveal themselves in an enhancement of some inelastic and nonlinear optical processes in photonic liquid crystals. As examples the corresponding experimentally observed effects for the enhancement of nonlinear optical second harmonic generation [31] and lowering of the lasing threshold [32] in photonic liquid crystals have to be mentioned along with the theoretically predicted enhancement of Cerenkov radiation (section 4 in [11] and chapter 5 in [12]).

In the conclusion should be stated that the results obtained here for the EM and DM (see also [27, 29] and [33, 34]) clarify the physics of these modes and manifests a complete agreement with the corresponding results of the previous investigations obtained by a numerical approach [13].

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